

**B022411(014)**

**B. Tech. (Fourth Semester) Examination,**

**Apri-May 2022**

**(CSE Branch)**

**(AICTE Scheme)**

**DISCRETE MATHEMATICS**

*Time Allowed : Three hours*

*Maximum Marks : 100*

*Minimum Pass Marks : 35*

*Note : Attempt all questions. Part (a) is compulsory from each unit & solve any two parts from (b), (c) & (d) of each unit.*

**Unit-I**

1. (a) Write short notes on Quantifier. 4
- (b) Define Boolean Algebra prove the following identities : 8

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- (i)  $a + a = a$   
(ii)  $a \cdot a = a$   
(iii)  $(a')' = a$
- (c) Explain the connectives conjunction, Disjunction, Conditional and Biconditional with truth tables. 8
- (d) Explain normal forms of Boolean functions change the following Boolean function to disjunctive normal form : 8

$$f(x, y, z) = \left[ x + (x' + y)' \right] \left[ x + (y' \cdot z')' \right]$$

### Unit-II

2. (a) Explain floor and ceiling functions with examples. 4
- (b) Define partial order relation. Prove that the relation "a divides b", it there exists an integer c such that  $ac = b$  and is denoted by  $a/b$ , on the set of all positive integers  $N$  is a partial order relation. 8
- (c) Define Injective, Surjective and Bijective function. If  $Q$  is the set of all rational numbers and  $f : Q \rightarrow Q$

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- is defined by  $f(x) = 3x + 2$ ,  $x \in Q$ , then prove that  $f$  is bijective function. Also find  $F^{-1}$ . 8
- (d) Define Lattice. Let  $L$  be the set of all factors of 12 and let "1" be the divisibility relation on  $L$ . Show that  $(L, '1')$  is a lattice. 8

### Unit-III

3. (a) Define group code. 4
- Show that  $(5, 2)$  encoding function  $e/B^2 \rightarrow B^5$  defined by
- $$e(00) = 00000 \quad e(01) = 01110$$
- $$e(10) = 10101 \quad e(11) = 11011$$
- is a group code.
- (b) Show that  $S = \{a + b\sqrt{2} : a, b \in z\}$  for the operations  $+$ ,  $\times$  is an integral domain but not a field. 8
- (c) State and prove Lagrange's theorem. 8

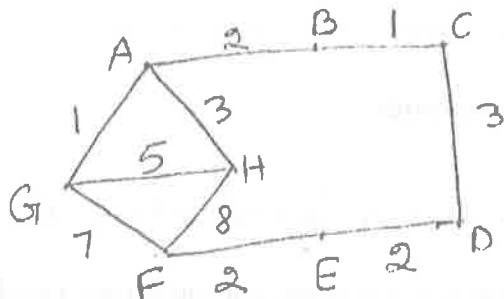
[ 4 ]

- (d) Define Isomorphism of groups. If  $G$  be the multiplicative group of three cube roots of unity,  $u$   
 $G = (\{1, w, w^2\}, \cdot)$  where  $w^3 = 1$  and  $G$  be the additive group of integers module 3  $u$ .  
 $G' = (\{0, 1, 2\}, +_3)$  then show that  $G$  and  $G'$  is isomorphic.

8

Unit-IV

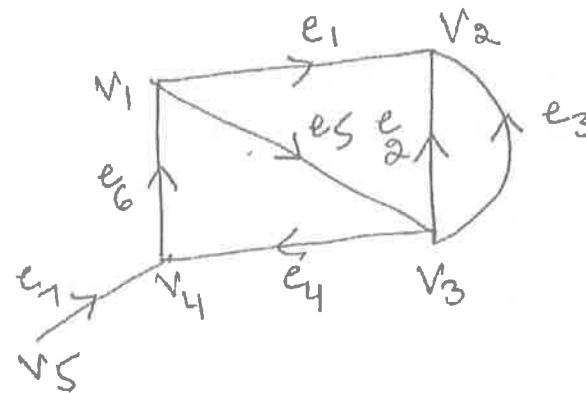
4. (a) Explain Walk, Path and Circuit is a graph with examples. 4  
 (b) Solve the travelling salesman problem for the following weighted graph. 8



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- (c) Define and find the incidence and adjacency matrix of the following diagram : 8



8

- (d) What is a simple graph. Show that the maximum number of edges in a simple graph with  $n$  vertices

$$\text{is } \frac{n(n-1)}{2}.$$

8

Unit-V

5. (a) Determine the generating function for the following sequence 4

$$a_r = \begin{cases} 2^r & \text{if } r \text{ is even} \\ -2^r & \text{if } r \text{ is odd} \end{cases}$$

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(b) Use mathematical induction to show that  $n! \geq 2^{n-1}$   
for  $n \geq 1$ . 8

(c) Find the number of integers between 1 and 250 that  
are divisible by any of the integers 2, 3, 5 and 7. 8

(d) Solve the difference equation

$$a_r - 4 a_{r-1} + 4 a_{r-2} = (r+1) 2^r \quad 8$$